

# A Wilkinson Power Divider with Different Power Ratios at Different Frequencies

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**Abstract**—The power divider/combiner is very important component for power amplifier design in low power MMIC design and high power system using hybrid scheme. In addition to that, the power divider/combiner is applied for impedance matching, antenna polarization, and phase control in phased array antenna. In this paper, we will consider the power divider/combiner working at two arbitrary frequencies with different power ratios at two output ports including a rigorous analysis and simulation. In order to determine the optimized design parameters, the even-odd mode analysis is carried out rigorously using impedance matching condition and lumped components such as R, L, and C. The validation of analysis and design parameters can be verified using two commercially available softwares based on circuit theory and FIT algorithm, respectively.

**Keywords**—Wilkinson power divider; eve-odd mode; unequal power divider; dual-band

## I. INTRODUCTION

As a predominant element used for combining and dividing the power in the microwave system, the Wilkinson power divider has been proposed by E. Wilkinson in 1960 [1]. This device has some advantages such as reciprocal characteristic and excellent isolation level between two output ports. Especially, in addition to the research on equal Wilkinson power divider resulting in dividing equal power into two ports, the dual-frequency Wilkinson power divider operating at two arbitrary frequencies (GSM and PCS) has been proposed recently [2].

In this paper, a Wilkinson power divider with different power ratios at different frequencies is suggested. A previous study in [3] shows an isolation problem resulting in imperfect isolation level between output ports. In order to overcome the isolation problem, passive components, R, L, and C are connected in parallel between two ports, port 2 and 3. In addition to that, two kinds of transmission lines are employed to satisfy the dual-frequency operation.

## II. EVEN-ODD MODE ANALYSIS

Consider a dual-frequency Wilkinson power divider with different power ratios as shown in Fig. 1. The lengths and characteristic impedances of transmission line are obtained from even-mode analysis whereas passive components, R, L,

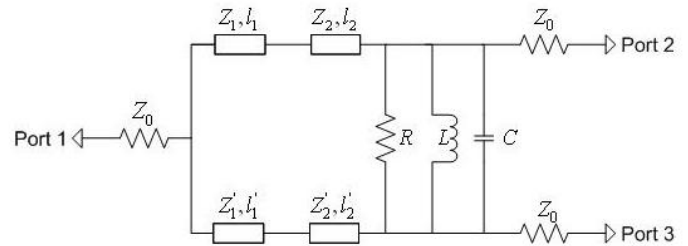


Figure 1. Unequal dual-frequency Wilkinson power divider

and C for improving isolation level are obtained from odd-mode analysis.

### A. Even-Mode Analysis

Since the voltages at port 2 and 3 are the same in even-mode and the current does not flow through the passive components, original structure of Fig. 1 is represented as Fig. 2(a) and (b). By introducing impedance matching at the middle point of two-section, both impedances looking into both directions at the interface point of two-section must be the same so that the reflection occurring at the interface is zero [4].

$$Z_2 \frac{Z_0 + jZ_2 \tan(-\beta l_2)}{Z_2 + jZ_0 \tan(-\beta l_2)} = Z_1 \frac{Z_a + jZ_1 \tan \beta l_1}{Z_1 + jZ_a \tan \beta l_1} \quad (1)$$

By separating eq. (1) into real and imaginary parts and comparing both sides, the following equation is obtained as

$$\tan \beta l_1 \tan \beta l_2 = \frac{Z_1 Z_2 (Z_a - Z_0)}{Z_2^2 Z_a - Z_1^2 Z_0} \quad (2-1)$$

$$\frac{\tan \beta l_1}{\tan \beta l_2} = \frac{Z_1 Z_0 Z_a - Z_1 Z_2^2}{Z_1^2 Z_2 - Z_2 Z_0 Z_a} \quad (2-2)$$

In the same way, the lengths and impedances of transmission line on the even-mode circuit of part 3 can be obtained as follow

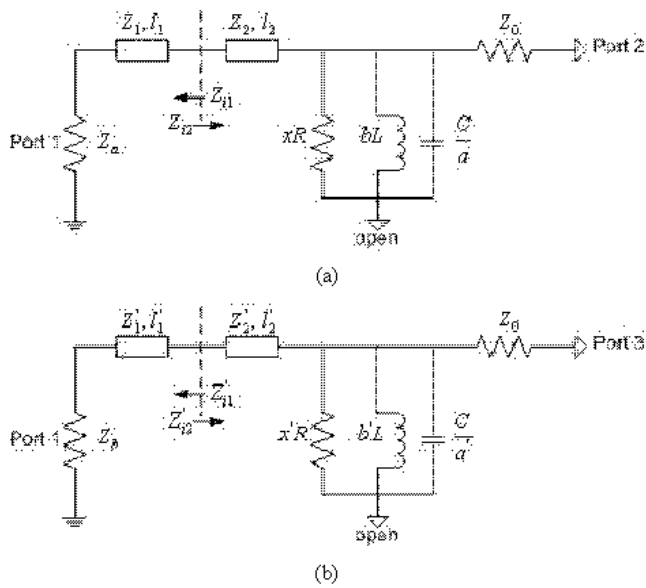


Figure 2. (a) Even-mode at port 2. (b) Even-mode at port 3.

$$\tan \beta l'_1 \tan \beta l'_2 = \frac{Z'_1 Z'_2 (Z_b - Z_0)}{Z_0^2 Z_b - Z_1^2 Z_0} \quad (3-1)$$

$$\frac{\tan \beta l'_1}{\tan \beta l'_2} = \frac{Z'_1 Z_0 Z_b - Z_1 Z_0^2}{Z_1^2 Z'_2 - Z_2 Z_0 Z_b} \quad (3-2)$$

The input impedances  $Z_a$  and  $Z_b$  are differently determined at different frequencies, respectively, to result in different power ratios at different frequencies since the power ratios at port 2 and 3 are dependent on the input impedance ratios. At this time, the ratio of  $Z_a$  and  $Z_b$ ,  $Z_a/Z_b$  must be equal to the characteristic impedance  $Z_0$ .

### B. Odd-Mode Analysis

In this case, the equivalent circuit can be divided into two parts at port 2 and 3 as shown in Fig. 3. The relationship between the unknowns  $x$ ,  $a$ , and  $b$  must satisfy the following conditions.

$$x + x' = 1, \quad a + a' = 1, \quad b + b' = 1 \quad (4)$$

The lumped elements,  $R$ ,  $L$ , and  $C$  contribute the electrical behaviors to matching the impedances seen looking into the left from port 2. The impedance  $Z_{io2}$  must be equal to  $Z_0$  to obtain a low-reflection coefficient.

$$Z_{io2} = \left( \frac{1}{Z_{io1}} + \frac{1}{xR} + j \left( \omega_1 \frac{C}{a} - \frac{1}{\omega_1 bL} \right) \right)^{-1} = Z_0 \quad (5)$$

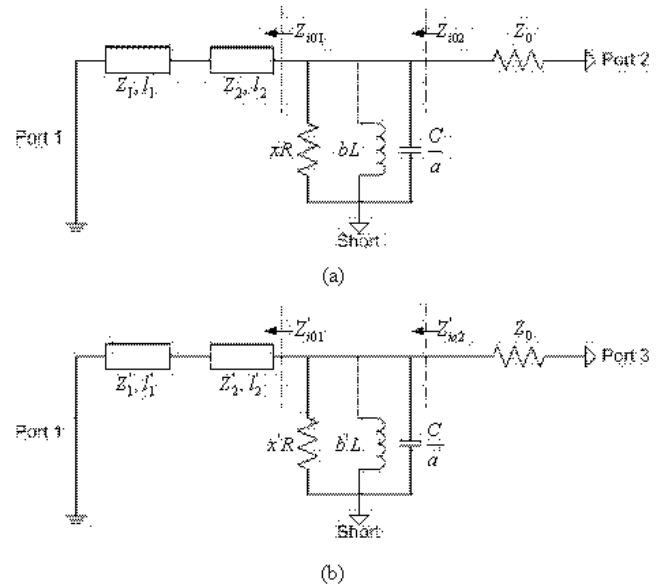


Figure 3. (a) Odd-mode at port 2. (b) Odd-mode at port 3.

By rearranging above equation, the lumped elements,  $R$ ,  $L$ , and  $C$  can be represented as

$$R = \frac{Z_0}{x}, \quad L = \frac{\omega_1}{\omega_2} \frac{b}{a} \frac{A - B}{\omega_2 A - \omega_1 B}, \quad C = \frac{A - B}{\omega_1} \frac{\omega_2}{\omega_2 a - \omega_1 a} \quad (6)$$

where

$$A = \frac{Z_2 - Z_1 \tan \beta l'_1 \tan \beta l'_2}{Z_2 (Z_1 \tan \beta l'_1 + Z_2 \tan \beta l'_2)}, \quad B = \frac{Z_2 - Z_1 \tan \beta_2 l_1 \tan \beta_2 l_2}{Z_2 (Z_1 \tan \beta_2 l_1 + Z_2 \tan \beta_2 l_2)} \quad (7)$$

In the same way,  $R$ ,  $L$ , and  $C$  components on the odd-mode circuit of port 3 can be obtained as follows

$$R = \frac{Z_0}{x'}, \quad L = \frac{\omega_1}{\omega_2} \frac{b'}{a'} \frac{A' - B'}{\omega_2 A' - \omega_1 B'}, \quad C = \frac{A' - B'}{\omega_1} \frac{\omega_2}{\omega_2 a' - \omega_1 a'} \quad (8)$$

where

$$A' = \frac{Z_2 - Z_1 \tan \beta_1 l'_1 \tan \beta_1 l'_2}{Z_2 (Z_1 \tan \beta_1 l'_1 + Z_2 \tan \beta_1 l'_2)}, \quad B' = \frac{Z_2 - Z_1 \tan \beta_2 l_1 \tan \beta_2 l_2}{Z_2 (Z_1 \tan \beta_2 l_1 + Z_2 \tan \beta_2 l_2)} \quad (9)$$

TABLE I. THE VALUE OF DESIGN PARAMETER

Frequency ( $f_1, f_2$ )	900 MHz	1800 MHz
Input impedance $Z_a$	83.3 $\Omega$	100 $\Omega$
Input impedance $Z_b$	125 $\Omega$	100 $\Omega$
$Z_1$	73.5 $\Omega$	
$Z_2$	60.5 $\Omega$	
$l_1$	22.5 mm	
$l_2$	29.7 mm	
$Z'_1$	87.9 $\Omega$	
$Z'_2$	66.2 $\Omega$	
$l'_1$	27 mm	
$l'_2$	21.6 mm	
R	100 $\Omega$	
L	15.5 nH	
C	0.96 pF	

### III. DESIGN PARAMETERS

In order to determine the lengths and characteristic impedances of each transmission line,  $Z_a$  and  $Z_b$  should be defined at first. When the power ratios between two ports at  $f_1 = 900$  MHz and  $f_2 = 1800$  MHz are required to be 1.5:1 and 1:1, respectively, the impedance ratios of  $Z_a$  and  $Z_b$  should satisfy 1:1.5 and 1:1, respectively. It is seen from the analysis that  $Z_a$  and  $Z_b$  are 83.3  $\Omega$  and 125  $\Omega$  at 900 MHz while  $Z_a$  and  $Z_b$  are 100  $\Omega$  and 100  $\Omega$  at 1800 MHz, respectively.

Thus, Eqs. (2) and (3) can be divided into Eqs. (9-1)~(9-4) and (10-1)~(10-4), respectively, corresponding to each target frequency.

$$\tan \beta_1 l_1 \tan \beta_1 l_2 = \frac{2Z_1 Z_2}{5Z_2^2 - 3Z_1^2} \quad (10-1)$$

$$\frac{\tan \beta_1 l_1}{\tan \beta_1 l_2} = \frac{Z_1 (5Z_0^2 - 3Z_2^2)}{Z_2 (3Z_1^2 - 5Z_0^2)} \quad (10-2)$$

$$\tan \beta_2 l_1 \tan \beta_2 l_2 = \frac{Z_1 Z_2}{2Z_2^2 - Z_1^2} \quad (10-3)$$

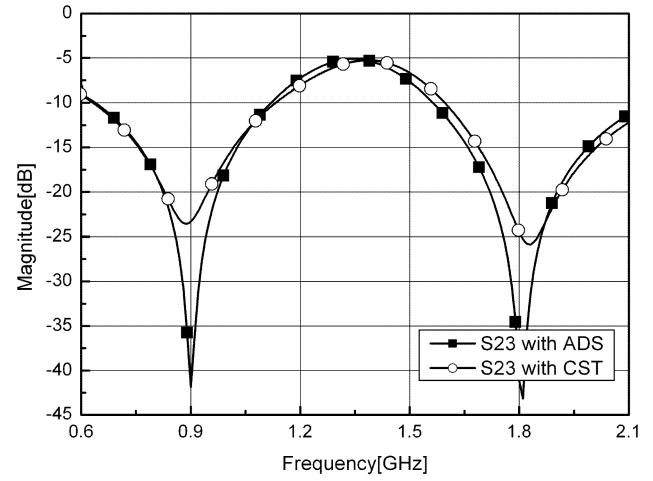
$$\frac{\tan \beta_2 l_1}{\tan \beta_2 l_2} = \frac{Z_1 (2Z_0^2 - Z_2^2)}{Z_2 (Z_1^2 - 2Z_0^2)} \quad (10-4)$$

$$\tan \beta_1 l'_1 \tan \beta_1 l'_2 = \frac{3Z'_1 Z'_2}{5Z_2'^2 - 2Z_1'^2} \quad (11-1)$$

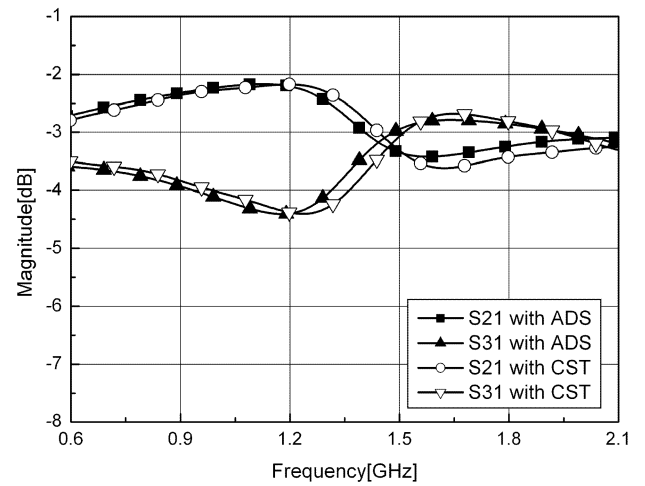
$$\frac{\tan \beta_1 l'_1}{\tan \beta_1 l'_2} = \frac{Z'_1 (5Z_0'^2 - 2Z_2'^2)}{Z'_2 (2Z_1'^2 - 5Z_0'^2)} \quad (11-2)$$

$$\tan \beta_2 l'_1 \tan \beta_2 l'_2 = \frac{Z'_1 Z'_2}{2Z_2'^2 - Z_1'^2} \quad (11-3)$$

$$\frac{\tan \beta_2 l'_1}{\tan \beta_2 l'_2} = \frac{Z'_1 (2Z_0'^2 - Z_2'^2)}{Z'_2 (Z_1'^2 - 2Z_0'^2)} \quad (11-4)$$



(a)



(b)

Figure 4. Simulation Results. (a) S23 (b) S21 and S31

The lengths and impedances of each transmission line can be evaluated and obtained from Eqs. (10) and (11). In addition to that, the passive components  $R$ ,  $L$ , and  $C$  satisfying the required isolation level can be obtained from Eqs. (6) and (8) and listed in Table I.

#### IV. SIMULATION RESULTS

Fig. 4 depicts the simulation results using the design parameters listed in Table I. A FR4 substrate having permittivity 4.9 and thickness 1.5 mm has been employed. In addition, two simulation tools such as circuit simulation (ADS) and EM simulation (CST MWS) have been used for optimization and design procedure. From the estimated results, it is seen from Fig. 4(b) that the power ratios of the proposed divider are 1.5:1 and 1:1 at  $f_1 = 900\text{MHz}$  and  $f_2 = 1800\text{MHz}$ , respectively, with high isolation level characteristics between two output ports.

#### V. CONCLUSION

A novel Wilkinson power divider with different power ratios at different frequencies has been proposed and designed using rigorous even-odd mode analysis. An addition of passive components,  $R$ ,  $L$ , and  $C$  in parallel gives good isolation level between output ports. It is also guaranteed that the proposed structure and analysis can be extended to a number of symmetrical passive structures requiring switching-effect even if the only passive components are employed.

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